2. RUBANOVSKII V.N., The relative equilibria of a gyrostat satellite, their bifurcation and stability. Prikl. Mat. Mekh., 52, 6, 1988.

Translated by D.L.

J. Appl. Maths Mechs, Vol. 55, No. 4, pp. 455-460, 1991
Printed in Great Britain

0021-8928/91 $15.00+0.00

STABILITY OF A SOLID CONTAINING A FLUID MOVING IN A FLUID*

P. CAPODANNO

France

(Received 3 December 1990)

A solid, suspended on a horizontal rod, with three pairwise orthogonal axes of symmetry which is placed in an ideal incompressible fluid executing a vortex-free motion is considered. The body has a cavity containing a fluid which is covered by an elastic membrane. Under certain conditions, the equations of motion of the system permit uniform translational motions of the whole system as a single body. The stability conditions for such motions are given.

1. Formulation of the problem. Let a solid S with three pairwise orthogonal axes of symmetry move in an ideal incompressible fluid of density ρ which is at rest at infinity. The body has a cavity containing an ideal fluid of density ρ' covered by an elastic membrane Σ of density ρ'', the contour of which, $\partial\Sigma$, is fixed onto the wall of the cavity. The "external fluid - body - internal fluid - membrane" system is located in a uniform gravitational field with an acceleration g.

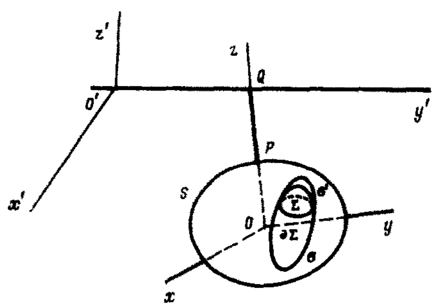

Fig.1

Let us now introduce three orthogonal coordinate systems: the inertial coordinate system $O'x'y'z'$ with the unit vectors $\mathbf{i}', \mathbf{j}', \mathbf{k}'$ and with the z'-axis directed along the ascending vertical, a moving $Oxyz$ coordinate system with the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, the axes of which coincide with the axes of symmetry of the body S, and the coordinate system ΩXYZ, the axes of which are parallel to the x-, y- and z-axes and the ΩXY plane contains the area Σ which is occupied by the membrane in the undeformed state. We shall assume that the body is suspended from a horizontal bar directed along the y'-axis using a solid rod PQ of negligibly small mass located along the z-axis and that $OP = a$ and $PQ = L$. We shall neglect the friction and action of the external fluid on the rod when the end of this rod Q moves along the axis of suspension (see Fig.1).

Let τ be the part of the cavity which is occupied by the fluid and let σ be the part of its wall which is wetted by the fluid. We will assume that the membrane is constantly in contact with the fluid and that the part of the cavity which is enclosed between the membrane

*Prikl. Matem. Mekhan., 55,4,572-577,1991

Σ and the remaining part of the wall σ' is filled with air at a constant pressure p_0. We will denote transverse displacements of points of the membrane by $w(X, Y, t)$.

Let μ_s, μ_f and μ_m be the mass of the body, of the internal fluid and of the membrane respectively and x_1, y_1, z_1; x_2, y_2, z_2 and x_{12}, y_{12} and z_{12} be the coordinates of the centres of gravity G_1, G_2 and G_{12} of the internal fluid, the membrane and the "internal fluid - membrane" system respectively. Finally, the central moments of inertia of the body S are denoted by A, B and C.

2. ***Equations of motion.*** We shall assume that the motion of the external fluid is vortex-free relative to the $O'x'y'z'$ coordinate system. There then exists a velocity potential $\Phi(x, y, z, t)$ which depends on the x, y, z coordinates of a fluid particle. By virtue of the condition for the slippage of the fluid over the surface of the body S, the potential can be represented in the form

$$\varphi = \sum_{i=1}^{3} (v_i\varphi_i + \omega_i\varphi_{3+i})$$

where v_1, v_2, v_3 and $\omega_1, \omega_2, \omega_3$ are the projections of the translational velocity vectors $\mathbf{v}$ (the velocity of the point O) and of the instantaneous angular velocity $\boldsymbol{\omega}$ of the body S on the x-, y- and z-axes while φ_i $(i = 1, \ldots, 6)$ are solely functions of x, y and z and they are harmonic in the domain occupied by the external fluid. These functions are solutions of well-known Neumann problems /1/.

The kinetic energy of the external fluid is finite and is given by the formula

$$T_f' = -\frac{1}{2}\rho \int_{\partial S} \frac{\partial\varphi}{\partial n} d\sigma$$

where $\partial\varphi/\partial n$ is the derivative with respect to φ along the direction of the external normal to the surface ∂S of the body S. Using T_f', it is possible to calculate the forces which are exerted by the fluid on the body S.

In the case under consideration, when the body S has three pairwise orthogonal axes of symmetry, we obtain the following expression for the kinetic energy T of the "body - external fluid" system /1/

$$2T = (\mu_s + \lambda_1) v_1^2 + (\mu_s + \lambda_2) v_2^2 + (\mu_s + \lambda_3) v_3^2 + (A + \lambda_4)\omega_1^2 +$$
$$(B + \lambda_5)\omega_2^2 + (C + \lambda_6)\omega_3^2, \quad \lambda_i = -\rho \int_{\partial S} \varphi_i \frac{\partial\varphi_i}{\partial n} d\sigma \quad (i = 1, \ldots, 6)$$

We will denote by $(\mathbf{R}_s, \mathbf{M}_s)$, $(\mathbf{R}_m, \mathbf{M}_m)$ and $(\mathbf{R}_f, \mathbf{M}_f)$ the principal vector and the principal moment with respect to the point O of the forces due to the pressure of the air between Σ and σ' on the body S, the tensile forces on the membrane which are distributed over the contour ∂S and the forces due to the pressure of the internal fluid while, we will denote by $\mathbf{R}$ the reaction on the rod at the point Q of the axis of suspension y' normal to y'. The equations of motion of the body S can then be written in the form

$$\frac{D}{Dt}(\text{grad}_v T) = -\mu_s g\mathbf{k}' + \mathbf{R}_s + \mathbf{R}_m + \mathbf{R}_f + \mathbf{R} \tag{2.1}$$
$$\frac{D}{Dt}(\text{grad}_w T) + \mathbf{v} \times \text{grad}_v T = \mathbf{M}_s + \mathbf{M}_m + \mathbf{M}_f + \mathbf{r}_Q \times \mathbf{R}$$

where $\mathbf{r}_Q$ is the radius vector of the point Q with respect to the point O.

The equations of motion of the "internal fluid - membrane" system have the form

$$(\mu_f + \mu_m)\frac{D^2\mathbf{r}_{12}'}{Dt^2} = -(\mu_s + \mu_m) g\mathbf{k}' - \mathbf{R}_s - \mathbf{R}_m - \mathbf{R}_f \tag{2.2}$$
$$\frac{D}{Dt}[\mathbf{K}_0 + \Theta\cdot\boldsymbol{\omega} + (\mu_f + \mu_m)\mathbf{r}_{12}' \times \mathbf{v}] + \mathbf{v} \times (\mu_f + \mu_m)\frac{D\mathbf{r}_{12}}{Dt} =$$
$$-\mathbf{r}_{12}' \times (\mu_f + \mu_m) g\mathbf{k}' - \mathbf{M}_s - \mathbf{M}_m - \mathbf{M}_f$$

where $\mathbf{r}_{12}'$ is the radius vector of the point G_{12} with respect to the point O', $\mathbf{K}_0$ is the kinetic moment with respect to point O of the "internal fluid - membrane" system in its motion with respect to the $Oxyz$ coordinate system and Θ_0 is its inertia tensor with respect to the point O.

By adding the corresponding Eqs.(2.1) and (2.2) term by term, denoting differentiation with respect to time in the coordinate system $Oxyz$ by d/dt and using the Coriolis theorem, we obtain

$$\frac{d}{dt}(\mathrm{grad}_v T) + \boldsymbol{\omega} \times \mathrm{grad}_v T + (\mu_f + \mu_m)\Big[\frac{d^2\mathbf{r}_{12}}{dt^2} + \frac{d\mathbf{v}}{dt} + \boldsymbol{\omega} \times \mathbf{v} + \tag{2.3}$$

$$\frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r}_{12} + \boldsymbol{\omega} \times \mathbf{r}_{12} + 2\boldsymbol{\omega} \times \frac{d\mathbf{r}_{12}}{dt}\Big] = -(\mu_s + \mu_f + \mu_m) g\mathbf{k}' + \mathbf{R}'$$

$$\frac{d}{dt}[\mathrm{grad}_\omega T + \mathbf{K}_0 + \Theta_0 \cdot \boldsymbol{\omega} + (\mu_f + \mu_m)\mathbf{r}_{12} \times \mathbf{v}] + \tag{2.4}$$

$$\boldsymbol{\omega} \times [\mathrm{grad}_\omega T + \mathbf{K}_0 + \Theta_0 \cdot \boldsymbol{\omega} + (\mu_f + \mu_m)\mathbf{r}_{12} \times \mathbf{v}] +$$

$$\mathbf{v} \times \Big[\mathrm{grad}_v T + (\mu_f + \mu_m)\Big(\frac{d\mathbf{r}_{12}}{dt} + \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}_{12}\Big)\Big] =$$

$$-\mathbf{r}_{12} \times (\mu_f + \mu_m)\mathbf{k}' + \mathbf{r}_{12} \times \mathbf{R}$$

We will write the equation for the transverse vibrations of the membrane in the form

$$\rho'' \frac{\partial^2 w}{\partial t^2} - T''\Delta w - p' - p_0 - \rho'' g\zeta_3 - \rho''[v_3^{\cdot} + \omega_1^{\cdot} y_0 - \omega_2^{\cdot} x_0 + \omega_1 v_2 - \tag{2.5}$$

$$\omega_2 v_1 + \omega_3(\omega_1 x_0 + \omega_2 y_0) - (\omega_1^2 + \omega_2^2) z_0 + \omega_1^{\cdot} Y - \omega_2^{\cdot} X +$$

$$\omega_3(\omega_1 X + \omega_2 Y) - (\omega_1^2 + \omega_2^2) w]$$

Here p' is the fluid pressure, T'' is the membrane tension, ζ_1, ζ_2 and ζ_3 are the cosines of the angles which are formed by the z'-axis with the x-, y-, z-axes, x_0, y_0 and z_0 are the coordinates of the point Ω and dots indicate differentiation with respect to time.

Denoting the velocity of a fluid particle relative to the axes of the $Oxyz$ coordinates by $\mathbf{u}$ and its radius vector with respect to point O by $\mathbf{r}$, we have the equation of motion of the fluid with the boundary conditions

$$\frac{d\mathbf{u}}{dt} + \frac{d\mathbf{v}}{dt} + \boldsymbol{\omega} \times \mathbf{v} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{u} =$$

$$-g\mathbf{k}' - \frac{1}{\rho^1}\mathrm{grad}\, p', \quad \mathrm{div}\,\mathbf{u} = 0 \quad \text{in} \quad \tau$$

$$\mathbf{u}\cdot\mathbf{n} = 0 \quad \text{on} \quad \sigma$$

$$u_1 \frac{\partial w}{\partial X} + u_2 \frac{\partial w}{\partial Y} - u_3 + \frac{\partial w}{\partial t} = 0 \quad \text{on} \quad \Sigma$$

Here $\mathbf{n}$ is a unit vector of the external, with respect to the domain τ, to the surface and u_1, u_2 and u_3 are the projections of the vector $\mathbf{u}$ on the x-, y- and z-axes. Terms of higher than first order with respect to the partial derivatives of the function w are discarded in the second condition.

The assumption regarding the attachment of the edge of the membrane to the wall of the cavity and the constancy of the volume of the fluid lead to the conditions

$$w = 0 \quad \text{on} \quad \partial\Sigma; \quad \int_{\Sigma_0} w\, d\sigma = 0$$

Let us now add the kinematic Poisson equation

$$\frac{d\mathbf{j}'}{dt} + \boldsymbol{\omega} \times \mathbf{j}' = 0, \qquad \frac{d\mathbf{k}'}{dt} + \boldsymbol{\omega} \times \mathbf{k}' = 0$$

and the condition of the orthogonality of the reaction $\mathbf{R}$ to the y'-axis

$$\mathbf{R}\cdot\mathbf{j}' = 0$$

to the equations which have been obtained above.

Finally, by differentiating the relationship $\mathbf{r}_0' = O'O\mathbf{j}' - (a + L)\mathbf{k}'$, where $\mathbf{r}_0'$ is the radius vector of point O relative to point O' and denoting the projections of the vector $\mathbf{j}'$ on the x-, y- and z-axes by η_1, η_2 and η_3, we get the equations

$$v_1 = \frac{dO'O}{dt}\eta_1 - (a + L)\omega_2, \quad v_2 = \frac{dO'O}{dt}\eta_2 + (a + L)\omega_1, \quad v_3 = \frac{dO'O}{dt}\eta_3$$

3. First integrals. By noting that the vectors $\mathbf{R}$ and $\mathbf{k}'$ are perpendicular to the y'-axis, we obtain the first integral

$$\Big[\mathrm{grad}_v T + (\mu_f + \mu_m)\Big(\frac{d\mathbf{r}_{12}}{dt} + \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}_{12}\Big)\Big]\cdot\mathbf{j}' = \mathrm{const} \tag{3.1}$$

from Eq.(2.3).

There is also an energy integral for the "external fluid - body"- internal fluid membrane system which, using T_f and T_m for the kinetic energies of the internal fluid and the membrane, can be written in the form

$$T + T_f + T_m = -\mu_s g \mathbf{r}_0' \cdot \mathbf{k}' - (\mu_f + \mu_m) r_{12}' k' - \frac{T''}{2} \int_{\Sigma_0} (w_X^2 + w_Y^2)\, d\sigma + \text{const} \tag{3.2}$$

4. Particular solution and transformation of the first integrals. Let us find the conditions for a uniform translational motion to exist in the direction of the y'-axis with a specified velocity v in which the x-, y- and z-axes are parallel to the x'-, y'- and z'-axes, the rod PQ is directed along the ascending vertical, the fluid and the membrane are at rest with respect to the body with the membrane in the undeformed state Σ_0. In this motion, we have

$$\eta_1 = 0,\ \eta_2 = 1,\ \eta_3 = 0,\ \zeta_1 = \zeta_2 = 0,\ \zeta_3 = 1 \tag{4.1}$$
$$v_1 = 0,\ v_2 = v,\ v_3 = 0,\ \omega_1 = \omega_2 = \omega_3 = 0,\ u \equiv 0,\ w \equiv 0$$

On introducing these values into the equations of motion, we conclude that $\mathbf{R} = (\mu_s + \mu_f + \mu_m) g\mathbf{k}'$, the vector $\mathbf{r}_{12}$ is parallel to z-axis and $p' = p_0 + \rho' g + \rho' g (z - z_0)$.

Hence, for the required motion to exist, it is necessary that the centre of gravity, G_{12}, of the "internal fluid - membrane" system, when the latter is in the undeformed state Σ_0, should lie on the z-axis. This condition is satisfied if, in the case of the cavity and the areas Σ_0, the planes $x = 0$ and $y = 0$ are planes of symmetry.

In order to investigate the stability of the above-mentioned motion we transform the first integrals by introducing into them velocities with respect to axes, which move uniformly and progressively at a velocity $\mathbf{v} = v\mathbf{j}'$.

We denote by u_1, u_2 and u_3 the projections of the velocity of a fluid particle or the membrane with respect to the $O'x'y'z'$ coordinate system and we put

$$v_1 = \bar{v}_1 + v\eta_1,\ v_2 = \bar{v}_2 + v\eta_2,\ v_3 = \bar{v}_3 + v\eta_3,$$
$$u_1 = \bar{u}_1 + v\eta_1,\ u_2 = \bar{u}_2 + v\eta_2,\ u_3 = \bar{u}_3 + v\eta_3,$$

where v_i and u_i $(i = 1, 2, 3)$ are the values of the corresponding variables in the perturbed motion. By expressing the kinetic energy of the "external fluid - body" system, of the internal fluid and the membrane, we can represent the first integrals in the form

$$\Phi_1 + \Phi_2 + \int_\tau F_1 \rho'\, d\tau + \int_\Sigma F_1 \rho''\, d\sigma = \text{const} \tag{4.2}$$

$$\frac{1}{2}[\Phi_3 + (A + \lambda_4)\omega_1^2 + (B + \lambda_5)\omega_2^2 + (C + \lambda_6)\omega_3^2 + 2v\Phi_1 + v^2\Phi_2] +$$
$$\frac{1}{2}\int_\tau F_2\rho'\, d\tau + v\int_\tau F_1\rho''\, d\tau + \frac{1}{2}\int_\Sigma F_2\rho''\, d\sigma + v\int_\Sigma F_1\rho''\, d\sigma -$$

$$(\mu_s + \mu_f + \mu_m) g (a + L)\zeta_3 + \int_\tau F_3\rho'\, d\tau + \int_\Sigma F_3\rho''\, d\sigma +$$
$$\frac{T''}{2}\int_{\Sigma_0} (w_X^2 + w_Y^2)\, d\sigma = \text{const}$$

$$\Phi_1 = \sum_{i=1}^{3} (\mu_s + \lambda_i)\bar{v}_i\eta_i,\quad \Phi_2 = \sum_{i=1}^{3} \lambda_i\eta_i^2,\quad \Phi_3 = \sum_{i=1}^{3} (\mu_s + \lambda_i)\bar{v}_i^2 \tag{4.3}$$
$$F_1 = \sum_{i=1}^{3} \bar{u}_i\eta_i,\quad F_2 = \sum_{i=1}^{3} \bar{u}_i^2,\quad F_3 = x\zeta_1 + y\zeta_2 + z\zeta_3$$

5. The problem of stability. By multiplying the integral (4.2) by v and subtracting it from (4.3), we obtain the first integral

$$E + W = \text{const} \tag{5.1}$$
$$2E = \Phi_3 + (A + \lambda_4)\omega_1^2 + (B + \lambda_5)\omega_2^2 + (C + \lambda_6)\omega_3^2 + \int_\tau F_2\rho'\, d\tau + \int_\Sigma F_2\rho''\, d\sigma$$
$$2W = v^2[(\lambda_2 - \lambda_1)\eta_1^2 + (\lambda_2 - \lambda_3)\eta_3^2] - 2(\mu_s + \mu_f + \mu_m) g (a + L)\zeta_3 +$$
$$2g\int_\tau F_3\rho'\, d\tau + 2g\int_\Sigma F_3\rho''\, d\sigma + T''\int_{\Sigma_0} (w_X^2 + w_Y^2)\, d\sigma_0$$

which will be used to study the stability of the motion (4.1).

The expression for E is a positive-definite functional which is solely dependent on the velocities, while W is a functional which is solely dependent on the position of the body S and the configuration of the fluid and the membrane. This makes it possible to use the stability theorem due to Rumyantsev /2/.

Let us denote the value of W in the unperturbed motion by W_0 and investigate the difference $W - W_0$. We first consider the difference

$$\int_{\tau}(x\zeta_1 + y\zeta_2 + z\zeta_3)\rho'\,d\tau - \int_{\tau_0} z\rho'\,d\tau,$$

where τ_0 is the domain which is occupied by the fluid in the unperturbed motion. We shall write it in the form

$$\int_{\tau-\tau_0}[x\zeta_1 + y\zeta_2 + (z_0 + Z)\zeta_3]\rho'\,d\tau + \int_{\tau_0}[x\zeta_1 + y\zeta_2 + z(\zeta_3 - 1)]\rho'\,d\tau$$

The first integral is calculated by successive integration. Noting that $\zeta_3 = 1 - {}^1/_2(\zeta_1{}^2 + \zeta_2{}^2) + \ldots$ and denoting the vector with the projections ζ_1, ζ_2 and $-{}^1/_2(\zeta_1{}^2 + \zeta_2{}^2)$ on the x-, y- and z-axes by Ξ, we obtain the expression

$$\int_{\Sigma_0}\left[(x\zeta_1 + y\zeta_2)w + \frac{w^2}{2}\right]\rho''\,d\sigma_0 + \mu_f\mathbf{r}_{10}\cdot\Xi + \ldots$$

for the difference under consideration.

Let us now consider the difference

$$\int_{\Sigma}(x\zeta_1 + y\zeta_2 + z\zeta_3)\rho''\,d\sigma - \int_{\Sigma_0} z\rho''\,d\sigma_0,$$

which we represent in the form

$$\frac{1}{2}z_0\int_{\Sigma_0}(w_X{}^2 + w_Y{}^2)\rho''\,d\sigma_0 + \mu_m\mathbf{r}_{20}\cdot\Xi + \ldots$$

Since $\mu_f\mathbf{r}_{10} + \mu_m\mathbf{r}_{20} = (\mu_f + \mu_m)(\mathbf{r}_{12})_0$ and it follows from the condition $\mathbf{j}'\cdot\mathbf{k}' = 0$ that $\zeta_2 = -\eta_3$ to a first approximation, we get, using the variables η_1, η_3 and ζ_1 that

$$2(W - W_0) = v^2(\lambda_2 - \lambda_1)\eta_1{}^2 + H\eta_3{}^2 + H'\zeta_1{}^2 + 2\rho'g\zeta_1\int_{\Sigma_0} xw\,d\sigma_0 -$$

$$2\rho'g\eta_3\int_{\Sigma_0} yw\,d\sigma_0 + \rho'g\int_{\Sigma_0} w^2\,d\sigma_0 + (T'' + \rho'gz_0)\int_{\Sigma}(w_X{}^2 + w_Y{}^2)\,d\sigma + \ldots$$

$$H = v^2(\lambda_2 - \lambda_3) + g\,[(\mu_s + \mu_f + \mu_m)(a + L) - (\mu_f + \mu_m)(z_{12})_0]$$

$$H' = H - v^2(\lambda_2 - \lambda_3)$$

where terms of higher than the second order of smallness in η_1, η_3, ζ_1, w, w_X and w_Y are denoted by the string of dots.

By taking account of the inequality

$$\Big(\int_{\Sigma_0} yw\,d\sigma_0\Big)^2 \leqslant I_{x_0}\int_{\Sigma_0} w^2\,d\sigma_0 \qquad \Big(\int_{\Sigma_0} xw\,d\sigma_0\Big)^2 \leqslant I_{y_0}\int_{\Sigma_0} w^2\,d\sigma_0,$$

where I_{x_0} and I_{y_0} are the moments of inertia about the x- and y-axes of the projection of the area Σ_0 on the $z = 0$ plane, and the inequality

$$\int_{\Sigma_0}(w_X{}^2 + w_Y{}^2)\,d\sigma_0 \geq \nu_0\int_{\Sigma_0} w^2\,d\sigma_0,$$

where ν_0 is the smallest eigenvalue of the boundary-value problem

$$\Delta w + \nu w = 0 \quad \text{on} \quad \Sigma_0;\ w = 0 \quad \text{on} \quad \partial\Sigma_0,$$

it is seen that the quadratic part ${}^1/_2\delta^2 W$ of the difference $W - W_0$ satisfies the inequality

$$\delta^2 W \geq v^2(\lambda_2-\lambda_1)\eta_1{}^2 + H\left(\eta_3 - \frac{\rho g}{H}\int_{\Sigma_0} yw\,d\sigma_0\right)^2 + H'\left(\zeta_1 + \frac{\rho' g}{H'}\int_{\Sigma_0} xw\,d\sigma_0\right)^2 +$$
$$\left[(T'' + \rho'' g z_0)\nu_0 + \rho' g\left(1 - \frac{\rho g I_{x_0}}{H} - \frac{\rho' g I_{y_0}}{H}\right)\right]\int_{\Sigma_0} w^2\,d\sigma_0$$

The coordinate (z_{12}) is, of course, less than $a + L$ and the constant H' is positive. On the other hand, the coefficient of the last term is positive if the tension T'' is sufficiently large. Hence, by virtue of Rumyantsev's theorem, the conditions: T'' is sufficiently large, $\lambda_2 > \lambda_1$ and

$$v^2(\lambda_2 - \lambda_3) + g\,[(\mu_s + \mu_f + \mu_m)(a + L) - (\mu_f + \mu_m)(z_{12})_0] > 0$$

are sufficient for the unperturbed motion (4.1) to be stable with respect to the parameters defining the position and velocity of the body S, to the norm $\| w \|_{L^2(\Sigma_0)}$ and the kinetic energy of the fluid and the membrane in their motion with respect to the body S.

Similar planar problems have been considered in /3-6/.

REFERENCES

1. KOCHIN N.E., KIBEL I.A. and ROZE N.V., Theoretical Hydrodynamics. 1, Fizmatgiz, Moscow, 1963.
2. RUMYANTSEV V.V., On the motion and stability of an elastic body with a cavity containing a fluid, Prikl. Matem. i Mekhan., 33, 6, 1969.
3. CAPODANNO P., Existence et stabilité des mouvements de translation horizontale rectiligne et uniforme d'un solide cylindrique pesant plongé dans un fluide parfait incompressible en mouvement plan irrotationel et possédant une cavité partiellement remplie de liquide pesant. J. Méc. 16, 1, 1977.
4. CAPODANNO P., Petites oscillations d'un liquide dans un vase fermé par une membrane élastique. ZAMP, 39, 6, 1988.
5. CAPODANNO P., Stability of a particular motion of a profile suspended by an elastic rod. Arch. of Mech. 40, 2-3, 1988.
6. CAPODANNO P., Sur les petites oscillations planes d'un liquide dans un recipient fixe fermé par un couvercle élastique. J. Mec. Theoriq. et Appl. 7, 1, 1988.

Translated by E.L.S.